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Total No. of Pages : 2

BT-1/D11 7501

Mathematics-I

Paper : MATH-101 E

e : Three Hours] [Maximum Marks : 100

e :– Attempt FIVE questions, selecting at least ONE question from each Unit.

UNIT–I

(a) Expand $\tan\left(\frac{\pi}{4}+x\right)$, by Taylore's series and hence find tan $(46°5°)$ correct to four decimal places.

(b) Show that the radius of curvature at an end of the major axis of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to the semi-latus rectum.

(a) Find the asymptotes of :

$(x + y)^2 (x + 2y + 2) = (x + 9y - 2)$.

(b) Trace the curve :

$y^2(x - a) = x^2(x + a)$.

UNIT–II

a) If $z = x\,\phi\left(\frac{y}{x}\right)+\psi\left(\frac{x}{y}\right)$, prove that, by using Euler's Theorem,

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=0\cdot$$

b) Transform the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}$ into polar coordinates.

Contd.

4. (a) In estimating the cost of a pile of bricks measured as $6m \times 50\,m \times 4m$. The tape is streched 1% beyond the standard length. If the count is 12 bricks in $1m^3$ and bricks cost Rs. 600/- per thousand. Find the approximate error in cost.

(b) Evaluate $\int_0^{\pi} \log(1 + \alpha \cos x)\, dx$, using the method of differentiation under the sign of integration.

UNIT–III

(a) Change the order of integration and then evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$.

(b) Find, by triple triangle, the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

(a) Calculate, by double integral, the volume generated by the revolution of the cardioid $r = a\,(1 - \cos\theta)$ about its axis.

(b) Express $\int_0^1 x^m (1 - x^n)^p\, dx$, in Terms of gamma function and evaluate $\int_0^1 x^5 (1 - x^2)^{10}\, dx$.

UNIT–IV

(a) Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1,3,–1).

(b) Prove that curl :

$$(\overline{F} \times \overline{G}) = \overline{F} \cdot \operatorname{div} \overline{G} - \overline{G} \operatorname{div} \overline{F} + (\overline{G} \cdot \nabla)\, \overline{F} - (\overline{F} \cdot \nabla)\, \overline{G}.$$

(a) Using Green's Theorem, evaluate :-

$\oint_C (y - \sin x)\, dx + \cos x\, dy]$, where C is the plane triangle enclosed by the line $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2}{\pi} x$.

Evaluate $\iint (x\,dy\,dz + y\,dz\,dx + z\,dx\,dy)$ of the surface of the sphere of radius a.